

## NUMERISCHE MATHEMATIK

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### FORMELN und TABELLEN

#### I. LINEARE GLEICHUNGSSYSTEME $Ax = b$

- *Direkte Verfahren: Cholesky-Zerlegung*

$$A = LL^T \Rightarrow LL^T x = b \Rightarrow 1) Ly = b; 2) L^T x = y$$

$$L = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}, \quad \text{wo} \quad l_{11} = \sqrt{a_{11}}; \quad l_{21} = \frac{a_{21}}{l_{11}}; \quad l_{31} = \frac{a_{31}}{l_{11}};$$

$$l_{22} = \sqrt{a_{22} - l_{21}^2}; \quad l_{32} = \frac{a_{32} - l_{31}l_{21}}{l_{22}}; \quad l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2};$$

- *Iterative Verfahren:*

Das Gesamtschrittverfahren

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{\substack{j=1 \\ j \neq i}}^n a_{ij} x_j^{(k)} \right), \quad i = 1, 2, \dots, n; \quad k = 0, 1, 2, \dots$$

Das Einzelschrittverfahren

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right), \quad i = 1, 2, \dots, n; \quad k = 0, 1, 2, \dots$$

#### II. INTERPOLATION DURCH POLYNOME *Die Lagrange-Polynome*

$$L_i(x) := \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j} = \frac{(x - x_0) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)}, \quad (i = 0, 1, \dots, n)$$

- Die Lagrangesche Interpolationsformel

$$P_n(x) = \sum_{i=0}^n y_i L_i(x);$$

$$P_4(x) = \sum_{i=0}^4 y_i L_i(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x);$$

- Die Newtonsche Interpolationsformel

$$P_n(x) = c_0 + \sum_{k=1}^n c_k (x - x_0)(x - x_1) \dots (x - x_{k-1}) =$$

$$= c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + c_3(x - x_0)(x - x_1)(x - x_2) + \dots + c_n(x - x_0)(x - x_1) \dots (x - x_{n-1})$$

$$c_k = [x_0; x_1; \dots; x_k], \quad (k = 0, 1, \dots, n), \quad c_0 = [x_0] = y_0.$$

- Die Rekursionsformel für  $c_k$

$$[x_0; x_1; \dots; x_k] = \frac{[x_1; x_2; \dots; x_k] - [x_0; x_1; \dots; x_{k-1}]}{x_k - x_0}, \quad (k = 1, \dots, n)$$

$$\text{mit } [x_{k-1}; x_k] = \frac{y_k - y_{k-1}}{x_k - x_{k-1}}, \quad (k = 1, \dots, n);$$

- Das Schema der dividierten Differenzen

| x     | y           | DD1                | DD2                     | DD3                          |
|-------|-------------|--------------------|-------------------------|------------------------------|
| $x_0$ | $y_0 = c_0$ |                    |                         |                              |
|       |             | $[x_0; x_1] = c_1$ |                         |                              |
| $x_1$ | $y_1$       |                    | $[x_0; x_1; x_2] = c_2$ |                              |
|       |             | $[x_1; x_2]$       |                         | $[x_0; x_1; x_2; x_3] = c_3$ |
| $x_2$ | $y_2$       |                    | $[x_1; x_2; x_3]$       |                              |
|       |             | $[x_2; x_3]$       |                         |                              |
| $x_3$ | $y_3$       |                    |                         |                              |

- Fehlerabschätzung

$$|f(x) - P_n(x)| \leq \frac{|f^{(n+1)}(\xi)|}{(n+1)!} |(x-x_0)(x-x_1)\dots(x-x_n)|;$$

$$|f(x) - P_n(x)| \leq \frac{M_{n+1}}{(n+1)!} |(x-x_0)(x-x_1)\dots(x-x_n)|, \quad \max_{\xi \in [x_0, x_n]} |f^{(n+1)}(\xi)| = M_{n+1}$$

- Die Newton-Gregory Interpolationsformel (Vorwärts Interpolationsformel)

$$P_n(x) = y_0 + \sum_{k=1}^n \binom{t}{k} \Delta^k y_0 = y_0 + \binom{t}{1} \Delta y_0 + \binom{t}{2} \Delta^2 y_0 + \dots + \binom{t}{n} \Delta^n y_0$$

mit  $t = \frac{x-x_0}{h}$  und

$$\Delta y_i = y_{i+1} - y_i, \quad (i=0,1,\dots,n-1), \quad \Delta^2 y_i = \Delta y_{i+1} - \Delta y_i, \quad (i=0,1,\dots,n-2)$$

$$\Delta^k y_i = \Delta^{k-1} y_{i+1} - \Delta^{k-1} y_i, \quad (k \geq 2) \quad \text{mit} \quad \nabla y_i = \nabla^1 y_i$$

Das Differenzenschema

| x     | y     | $\Delta$     | $\Delta^2$     | $\Delta^3$     | $\Delta^4$     |
|-------|-------|--------------|----------------|----------------|----------------|
| $x_0$ | $y_0$ |              |                |                |                |
|       |       | $\Delta y_0$ |                |                |                |
| $x_1$ | $y_1$ |              | $\Delta^2 y_0$ |                |                |
|       |       | $\Delta y_1$ |                | $\Delta^3 y_0$ |                |
| $x_2$ | $y_2$ |              | $\Delta^2 y_1$ |                | $\Delta^4 y_0$ |
|       |       | $\Delta y_2$ |                | $\Delta^3 y_1$ |                |
| $x_3$ | $y_3$ |              | $\Delta^2 y_2$ |                |                |
|       |       | $\Delta y_3$ |                |                |                |
| $x_4$ | $y_4$ |              |                |                |                |

- Spline-Interpolation (Vorlesung 7)

- Funktionsapproximation: Methode der kleinsten Quadraten (Vorlesung 7)