NUMERISCHE MATHEMATIK

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FORMELN und TABELLEN

I. LINEARE GLEICHUNGSSYSTEME $Ax = b$

- *Direkte Verfahren: Cholesky-Zerlegung*

$$A = L L^T \quad \Rightarrow \quad LL^T x = b \quad \Rightarrow \quad 1)\ Ly = b; \quad 2)\ L^T x = y$$

$$L = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}, \qquad \text{wo} \qquad l_{11} = \sqrt{a_{11}}\ ; \quad l_{21} = \frac{a_{21}}{l_{11}}; \quad l_{31} = \frac{a_{31}}{l_{11}};$$

$$l_{22} = \sqrt{a_{22} - l_{21}^2}\ ; \quad l_{32} = \frac{a_{32} - l_{31} l_{21}}{l_{22}}; \quad l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2}\ ;$$

- *Iterative Verfahren:*

Das Gesamtschrittverfahren

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j^{(k)} \right), \ i = 1, 2, \ldots, n; \ k = 0, 1, 2, \ldots$$

Das Einzelschrittverfahren

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right), \ i = 1, 2, \ldots, n; \ k = 0, 1, 2, \ldots$$

II. INTERPOLATION DURCH POLYNOME *Die Lagrange-Polynome*

$$L_i(x) := \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j} = \frac{(x - x_0) \ldots (x - x_{i-1})(x - x_{i+1}) \ldots (x - x_n)}{(x_i - x_0) \ldots (x_i - x_{i-1})(x_i - x_{i+1}) \ldots (x_i - x_n)}, \ (i = 0, 1, \ldots, n)$$

- *Die Lagrangesche Intepolationformel*

$$P_n(x) = \sum_{i=0}^{n} y_i L_i(x) \ ;$$

$$P_4(x) = \sum_{i=0}^{4} y_i L_i(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L_4(x) \ ;$$

- *Die Newtonsche Interpolationsformel*

$$P_n(x) = c_0 + \sum_{k=1}^{n} c_k (x - x_0)(x - x_1)\dots(x - x_{k-1}) =$$

$$= c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + c_3(x - x_0)(x - x_1)(x - x_2) + \dots$$

$$\dots + c_n(x - x_0)(x - x_1)\dots(x - x_{n-1})$$

$$c_k = [x_0; x_1; \dots; x_k], \quad (k = 0, 1, \dots, n), \qquad c_0 = [x_0] = y_0 .$$

- *Die Rekursionformel für* c_k

$$[x_0; x_1; \dots; x_k] = \frac{[x_1; x_2; \dots; x_k] - [x_0; x_1; \dots; x_{k-1}]}{x_k - x_0}, \quad (k = 1, \dots, n)$$

mit $[x_{k-1}; x_k] = \dfrac{y_k - y_{k-1}}{x_k - x_{k-1}}, \quad (k = 1, \dots, n);$

- *Das Shema der dividierten Differenzen*

x	y	DD1	DD2	DD3
x_0	$y_0 = c_0$			
		$[x_0; x_1] = c_1$		
x_1	y_1		$[x_0; x_1; x_2] = c_2$	
		$[x_1; x_2]$		$[x_0; x_1; x_2; x_3] = c_3$
x_2	y_2		$[x_1; x_2; x_3]$	
		$[x_2; x_3]$		
x_3	y_3			

- *Fehlerabschätzung*

$$\left|f(x)-P_n(x)\right| \le \frac{\left|f^{(n+1)}(\xi)\right|}{(n+1)!}\left|(x-x_0)(x-x_1)\ldots(x-x_n)\right|;$$

$$\left|f(x)-P_n(x)\right| \le \frac{M_{n+1}}{(n+1)!}\left|(x-x_0)(x-x_1)\ldots(x-x_n)\right|, \qquad \max_{\xi\in[x_0,x_n]}\left|f^{(n+1)}(\xi)\right| = M_{n+1}$$

- *Die Newton-Gregory Interpolationsformel (VorwärtsInterpolationsformel)*

$$P_n(x)= y_0+\sum_{k=1}^{n}\binom{t}{k}\Delta^k y_0 = y_0+\binom{t}{1}\Delta y_0+\binom{t}{2}\Delta^2 y_0+\ldots+\binom{t}{n}\Delta^n y_0$$

mit $t=\dfrac{x-x_0}{h}$ und

$\Delta y_i = y_{i+1}-y_i$, $\left(i=0,1,\ldots,n-1\right)$, $\Delta^2 y_i=\Delta y_{i+1}-\Delta y_i$, $\left(i=0,1,\ldots,n-2\right)$

$\Delta^k y_i=\Delta^{k-1}y_{i+1}-\Delta^{k-1}y_i$, $\left(k\ge 2\right)$ mit $\nabla y_i=\nabla^1 y_i$

Das Differenzenschema

x	y	Δ	Δ^2	Δ^3	Δ^4
x_0	y_0				
		Δy_0			
x_1	y_1		$\Delta^2 y_0$		
		Δy_1 ,		$\Delta^3 y_0$	
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$
		Δy_2		$\Delta^3 y_1$	
x_3	y_3		$\Delta^2 y_2$		
		Δy_3			
x_4	y_4				

- *Spline-Interpolation (Vorlezung 7)*

- *Funktionsapproximation: Methode der kleinsten Quadraten (Vorlezung 7)*