Faculty of German Engineering and Industrial Management Education - FDIBA

# Introduction to Computer Graphics

## Transformations

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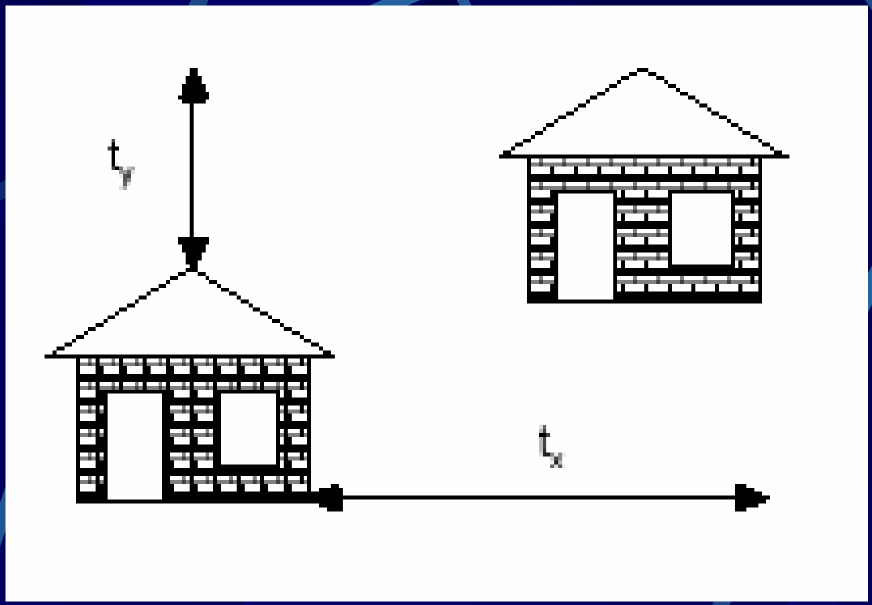
# Transformations

- **Basic Transformations**
  - Translation
  - Scale
  - Rotation
- **Compound Transformations**

# Translation

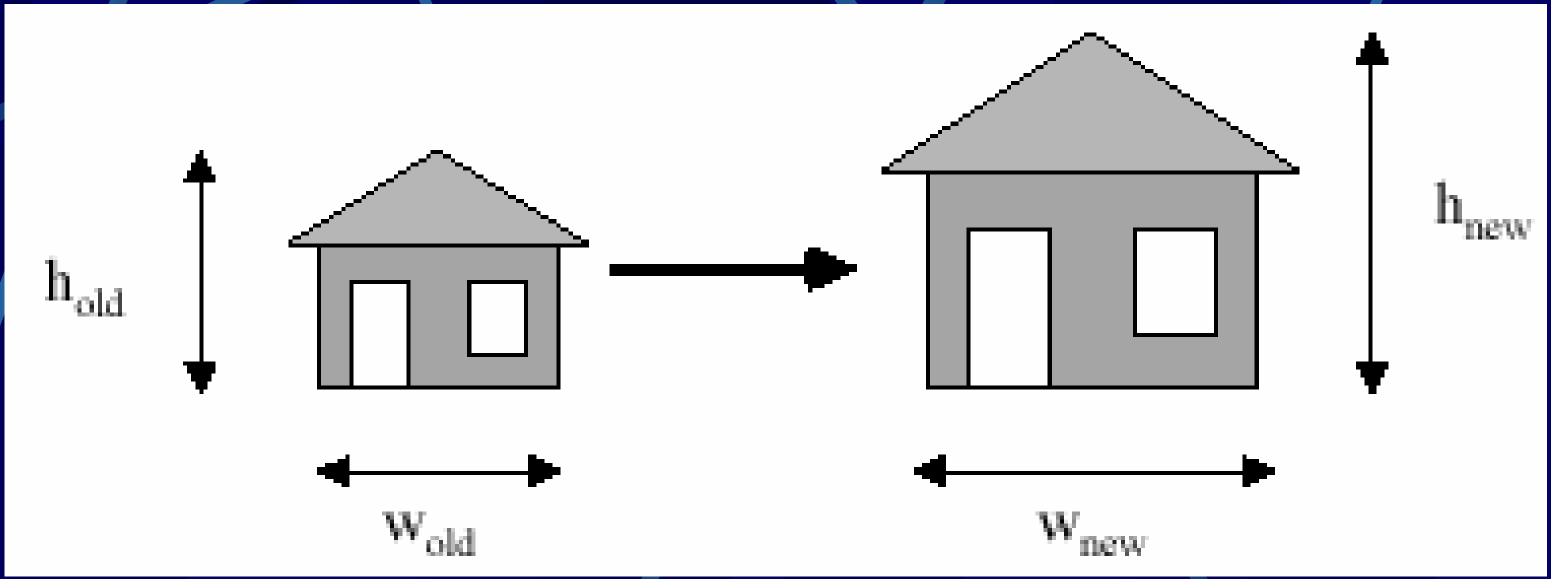



$x_{new} = x_{old} + d_x$ ;
$y_{new} = y_{old} + d_y$

- We translate an object by translating each vertex in the object.
- Add 2 vectors $\begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \end{bmatrix}$

# Scale

- About the **origin**!

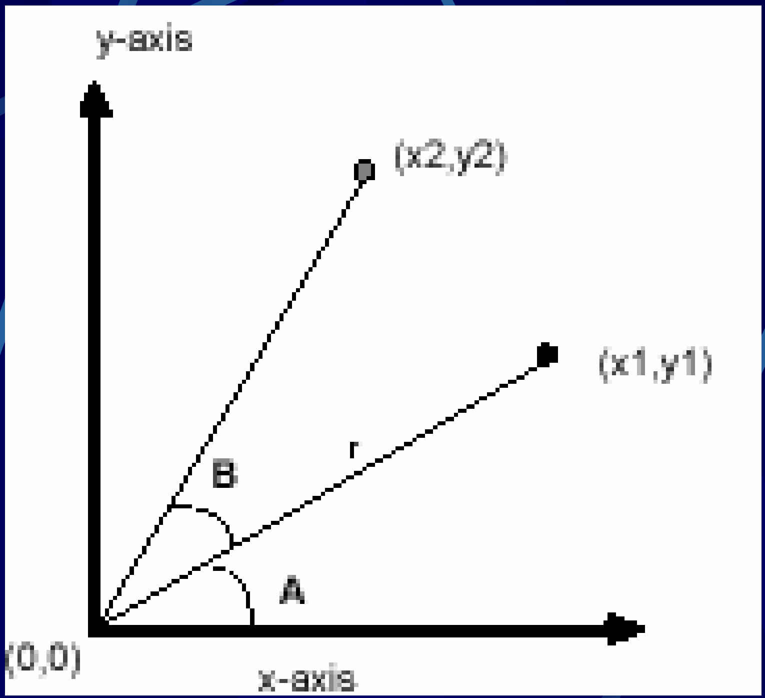



- $s_x = w_{new} / w_{old}$ ; $s_y = h_{new} / h_{old}$
- $x_{new} = s_x \, x_{old}$; $y_{new} = s_y \, y_{old}$
- Multiply matrix by vector

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x \cdot x \\ s_y \cdot y \end{bmatrix}$$

# Rotation

- About the origin!




$$x_2 = x_1 \cos\theta - y_1 \sin\theta$$
$$y_2 = x_1 \sin\theta + y_1 \cos\theta$$

- Multiply matrix by vector

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

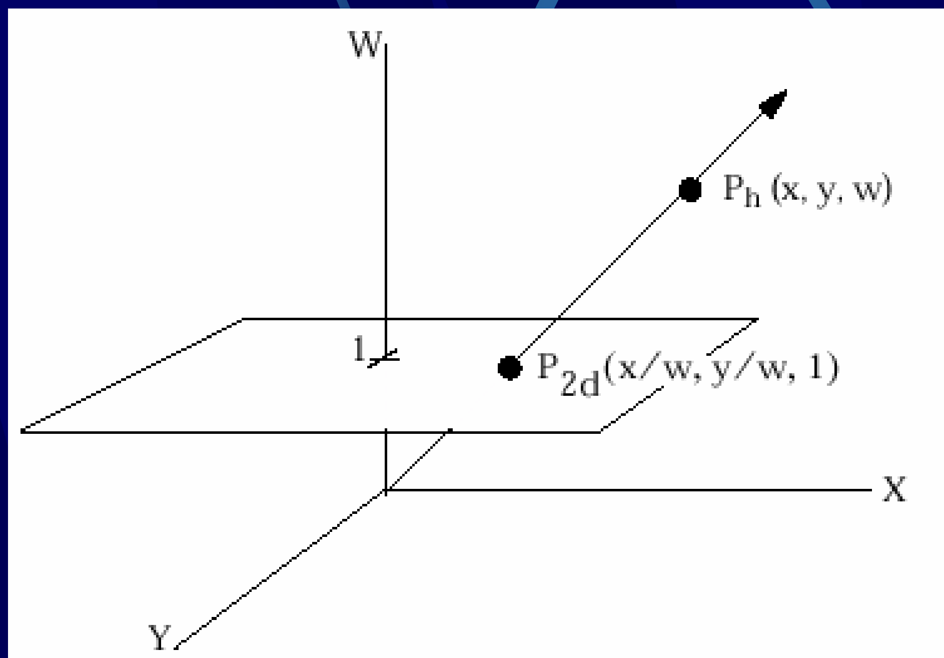
# Look for a common operation

- Describe T, S, R with operation of the same structure:
- Multiply matrix with vector

- Introduce a new artificial coordinate:
- Homogeneous Coordinates
- Point in 2D space --> 3 Coordinates

# Homogeneous Coordinates

*What is* $\begin{bmatrix} x \\ y \\ w \end{bmatrix}$ *?*



# Transformations in Homogeneous Coordinates

- Points in homogeneous coordinates:
- p’ = M . P, where M =

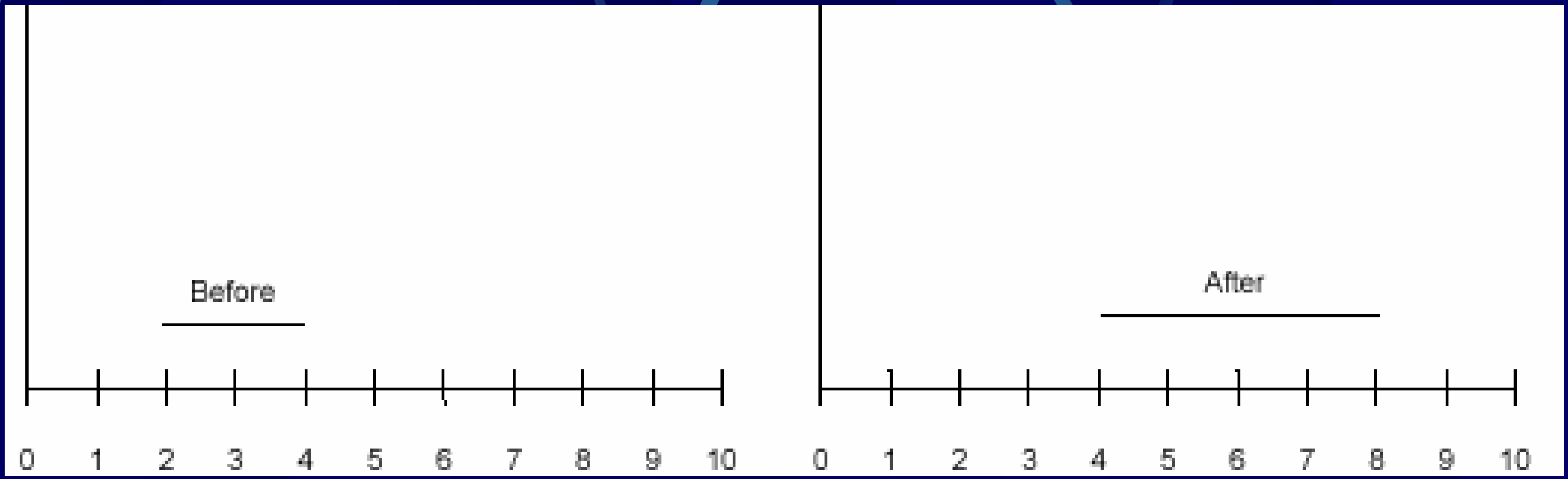
$$T(dx, dy) = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}$$

$$S(s_x, s_y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

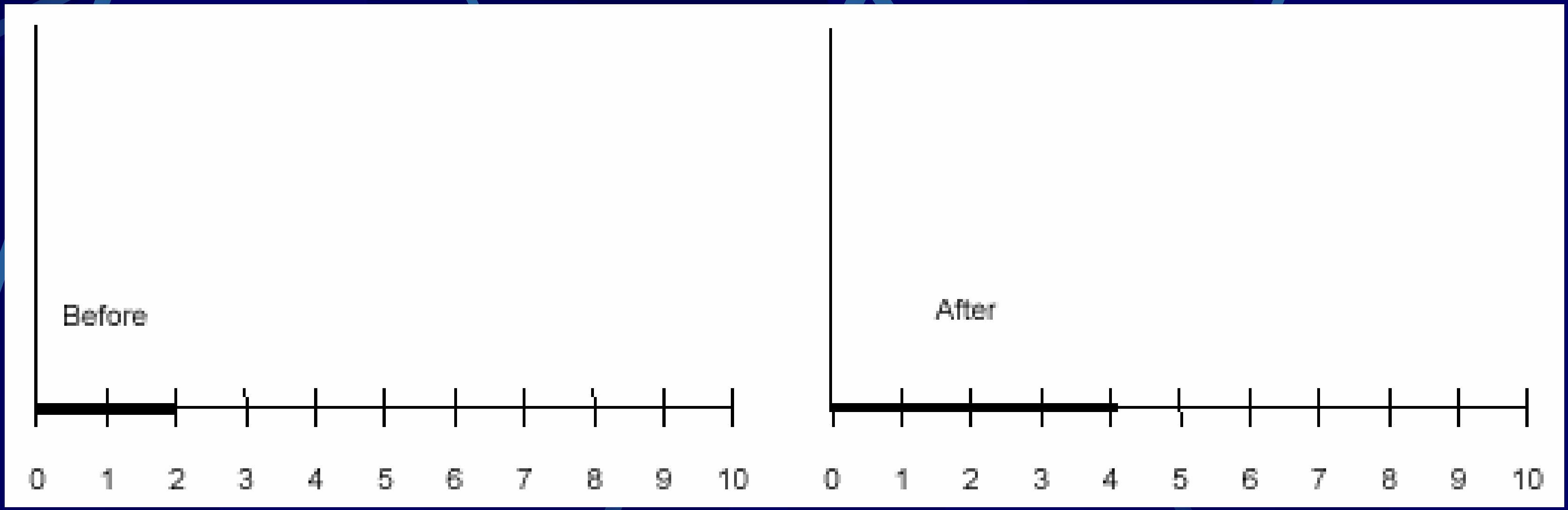
# Composite Transformations

- Example: Scale an object about a given point (not the origin)
  - A problem with the scale transformation is that it also moves the object being scaled.
  - Scale a line between (2, 1) (4,1) to twice its length.



# Composite Transform. - Scale

- If we scale a line between (0,0) & (2,0) to twice its length, the left-hand endpoint does not move.



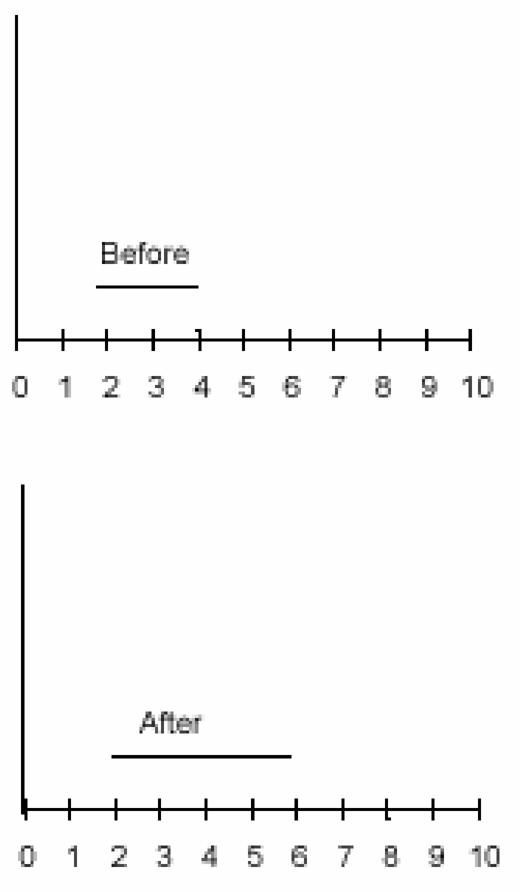

- (0,0) is known as a **fixed point** for the basic scaling transformation.
- We can use **composite transformations** to create a scale transformation with different fixed points.

# Composite Transform . – Scale (2)

- Scale by 2 with **fixed point** = (2,1)
  - Translate the point (2,1) to the origin
  - Scale by 2
  - Translate origin to point (2,1)

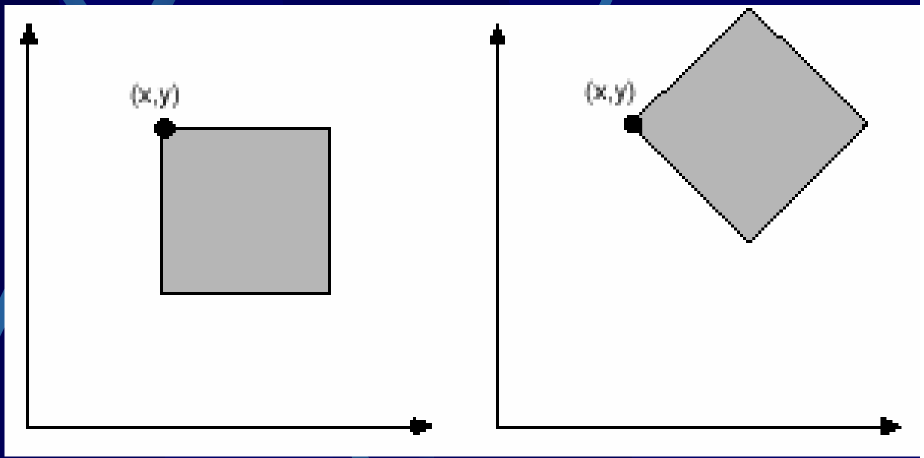
$$\underbrace{\begin{bmatrix}1&0&2\\0&1&1\\0&0&1\end{bmatrix}}_{T_{2,1}}\underbrace{\begin{bmatrix}2&0&0\\0&1&0\\0&0&1\end{bmatrix}}_{S_{2,1}}\underbrace{\begin{bmatrix}1&0&-2\\0&1&-1\\0&0&1\end{bmatrix}}_{T_{-2,-1}}=\underbrace{\begin{bmatrix}2&0&-2\\0&1&0\\0&0&1\end{bmatrix}}_{C}$$

$$\underbrace{\begin{bmatrix}2&0&-2\\0&1&0\\0&0&1\end{bmatrix}}_{C}\begin{bmatrix}2\\1\\1\end{bmatrix}=\begin{bmatrix}2\\1\\1\end{bmatrix}\qquad\underbrace{\begin{bmatrix}2&0&-2\\0&1&0\\0&0&1\end{bmatrix}}_{C}\begin{bmatrix}4\\1\\1\end{bmatrix}=\begin{bmatrix}6\\1\\1\end{bmatrix}$$



# Composite Transform. - Rotate

- Rotation of θ Degrees About Point (x,y)
  - Translate (x,y) to origin
  - Rotate
  - Translate origin to (x,y)

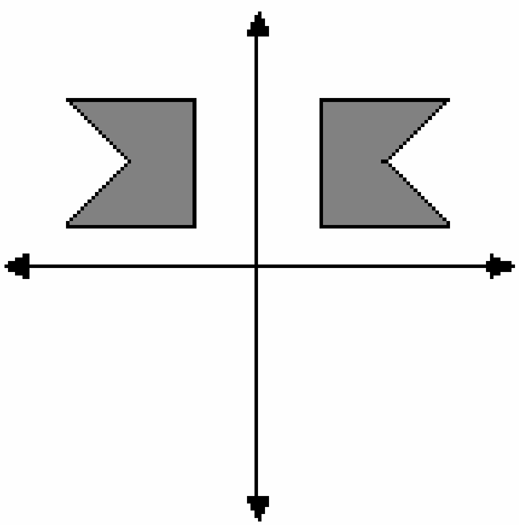



$$C = \underbrace{\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix}}_{T_{x,y}} \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_\theta} \underbrace{\begin{bmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{bmatrix}}_{T_{-x,-y}}$$

# Reflections

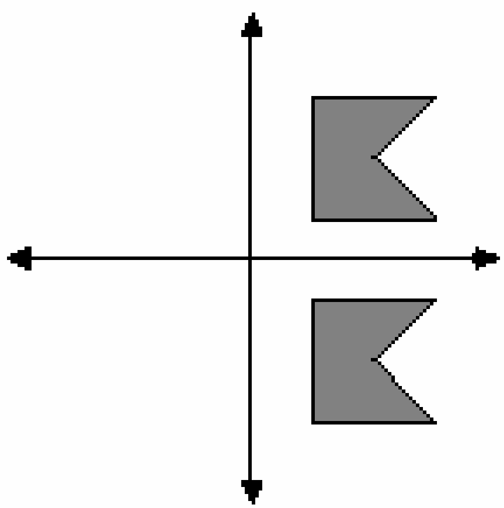
Reflection about the y-axis

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflection about the x-axis

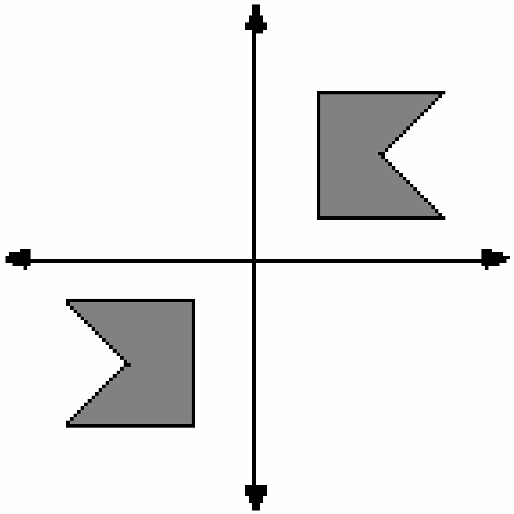
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



# Reflections (2)

Reflection about the origin

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Reflection about the line y=x

?

?

# Transformations

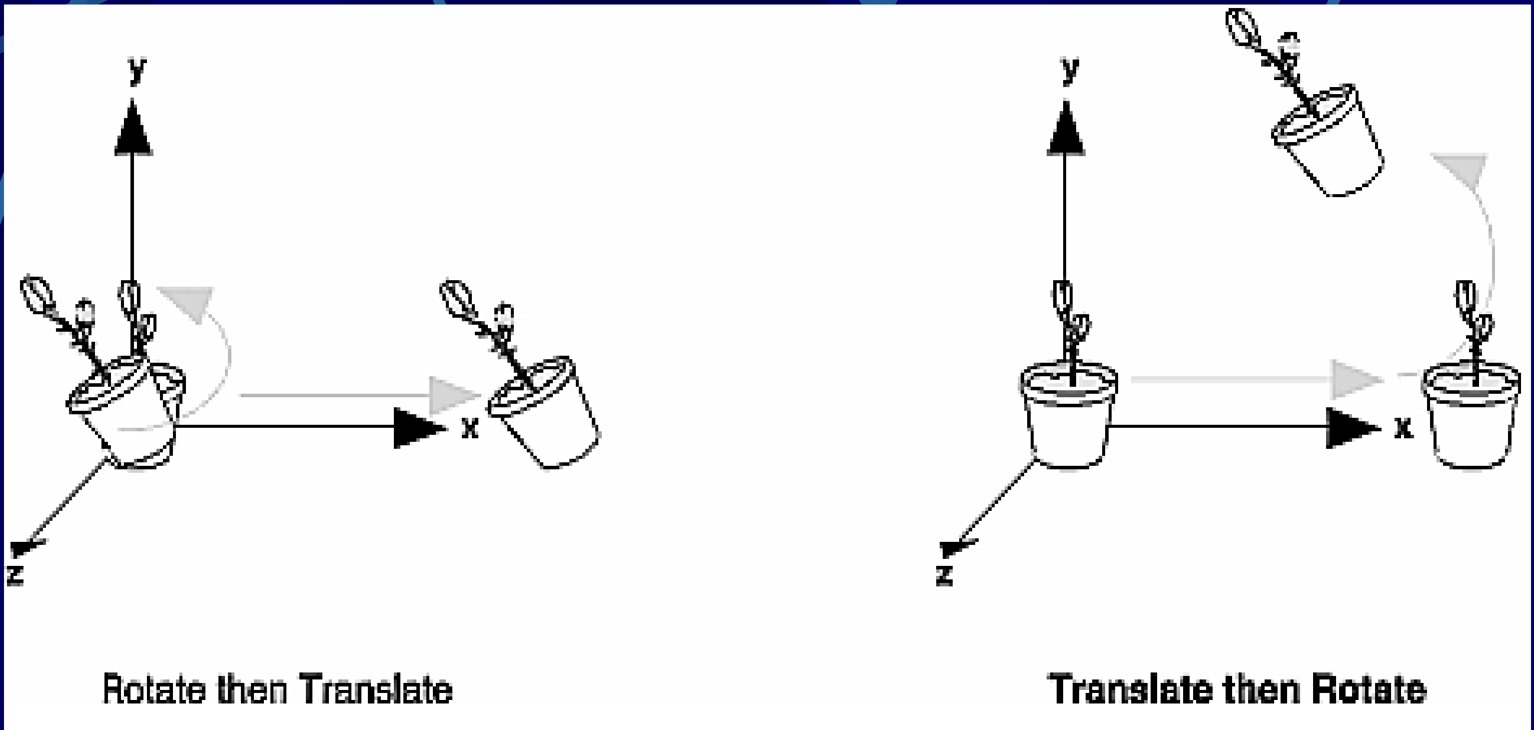
- General Form of matrix M

- Affine Transformations: preserve the proportions and the parallelism of lines

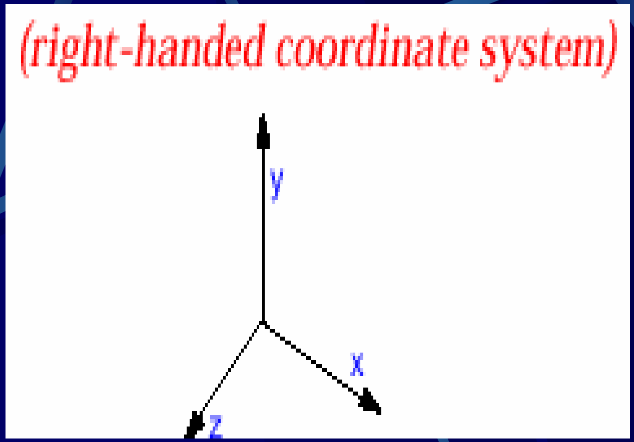
- Euclid transformations: preserve the distances and the angles of lines -> rigid body transformations: R, T

# Transformations

- The order of transformations is importantl.



# 3D Basic Transformations




- Translation

$$\begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Scaling

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 3D Basic Transformations

- Rotation about Z-axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about X-axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotation about Y-axis

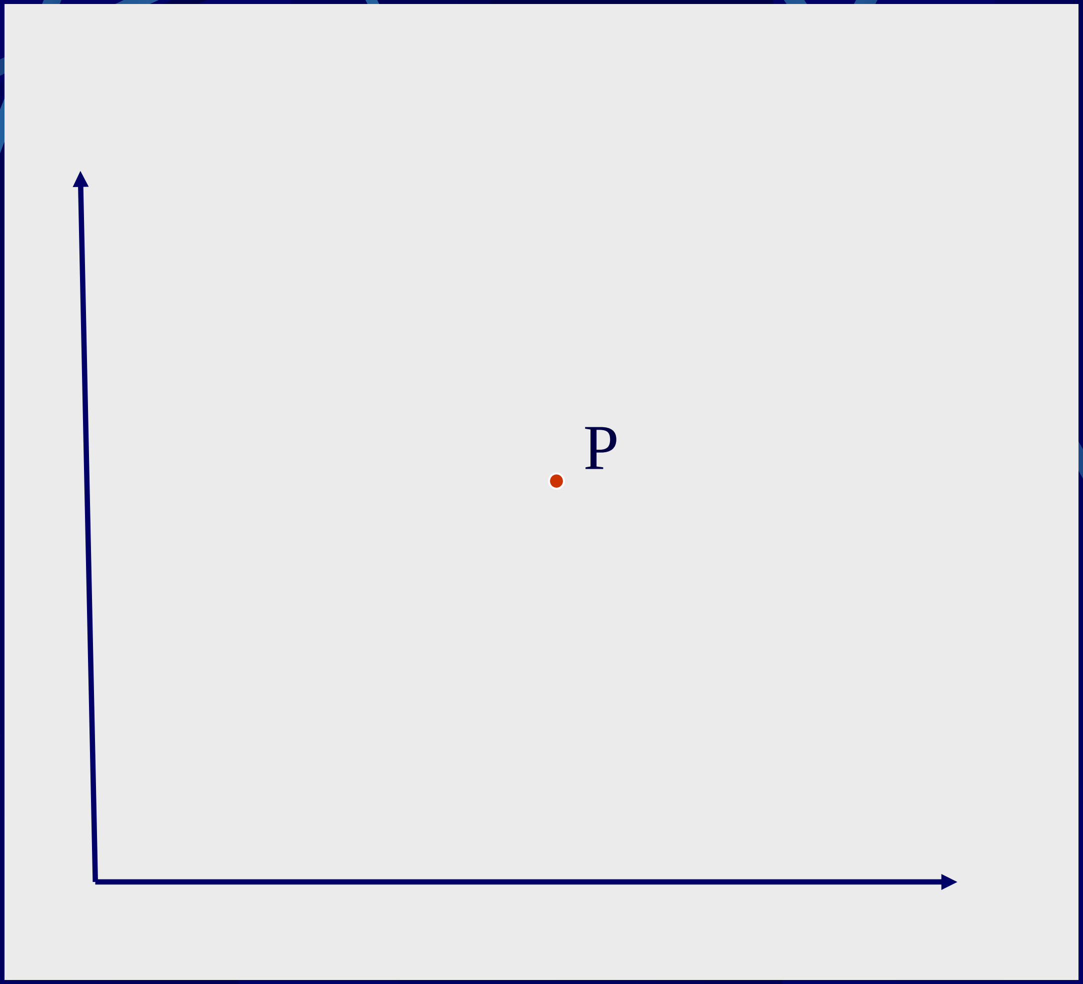
$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# Change the coordinate system

- Two approaches:
- Transform objects according to one coordinate system
  - or
- Leave objects unchanged and change the coordinate system -> the reference of the description
- Example

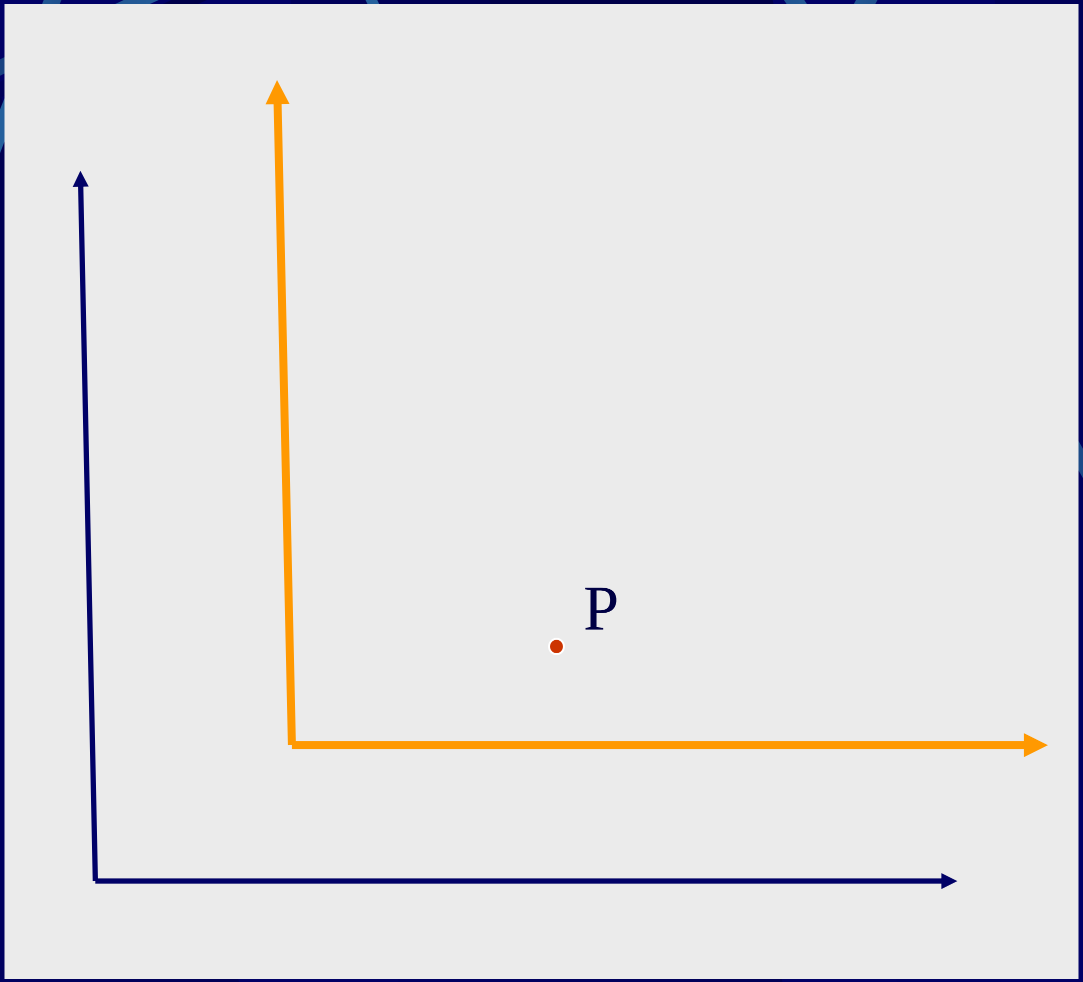
# Change the coordinate system

Example



# Change the coordinate system

Example



# Change the coordinate system

Example